Mastering Word Problems

# A HOME STUDY GUIDE For Those Who Love Solving Challenging Math Problems 

This Guide has 12 pages. Pages 1-3: Instructions, Flowchart and Checklist. Pages 4-11 are a reproduction of Instructional Guide for Problem \#9 from Teaching Mathematical Problem Solving (the $B Y B$ ). Page 12 is details about the Big Yellow Book and order information.

| Instructions for Parents / Coaches | Instructions for Students / Mathletes |
| :---: | :---: |
| - Home study requires parent involvement in directing their child's growth in the area of mathematical problem solving. <br> - You do not need to be mathematically strong to support your child's learning. <br> - The emphasis of Math Olympiad problems is on math thinking skills, not computation skills. <br> - You should encourage your child to verbally explain the material presented in the book to you. Your role is to listen and ask questions. Mastering problem solving requires patience and perseverance. <br> - To maximize benefit, follow the study sequence shown in the Problem Flowchart (Guide Page 2). <br> - The study sequence is illustrated by Problem \#9 from the $B Y B$. The pages listed in the Flowchart are reproduced in this packet. <br> - Your child should do the activity on each $B Y B$ page in the order they are on the Flowchart. <br> - As you proceed through the $B Y B$, allocate more time to do each problem. <br> - Encourage your child to take as much time as needed for each problem, focusing on understanding not speed. <br> - Encourage your child to do work from the $B Y B$ over an extended period of time. They may want to do some problems, leave the book for a while and return at another time. <br> - Each year your child's mathematical knowledge and experience increases. Encourage your child to return to the $B Y B$ over the years as their ability to do and understand the problems will change over time. <br> - For more instructional background and to view the author's videos go to www.moems.org/TMPS. | - Mastering problem solving takes time and thought. It is not how fast you go, but how much you understand about solving problems. <br> - The Problem Flowchart (Guide Page 2) provides the instructional steps that you should follow to master problem solving. <br> - Go over the Flowchart with a parent to learn the instructional steps. Follow these steps with all $B Y B$ problems. <br> - The Flowchart uses $B Y B$ Problem \#9 to illustrates the steps. Do the Problem \#9 after you read all these instructions. <br> - Use the BYB Problem Checklist (Guide Page 3) to keep track of your progress in mastering word problems. <br> - To master word problems, we recommend you do ALL the problems in the $B Y B$ in order. The problems are arranged so that they become more complex as you proceed. <br> - The Flowchart was designed to help you improve your problem solving. You will not get a grade for your work. This is about you getting better. Set your own pace and use the material in the $B Y B$ to help you improve. <br> - Now do $B Y B$ Problem \#9 using the BYB Problem Flowchart. <br> - Have FUN!!! |

## BYB Problem Flowchart <br> BYB = Big Yellow Book <br> Using BYB Problem \#9 as an Example



## BYB Problem Checklist

$B Y B=$ Big Yellow Book

Instructions: The page numbers in the table refer to pages in the $B Y B$.
Use the $B Y B$ Problem Flowchart to guide your work.
Cross each page number out as you complete the activity for that page.

| $\begin{gathered} B Y B \\ \text { Problem } \\ \# \end{gathered}$ | Problem Page | Answer Page | Solution <br> Strategy Page | Questions to Extend Learning | Questions <br> to Get <br> Unstuck <br> or Started |
| :---: | :---: | :---: | :---: | :---: | :---: |
| \#1 | 99 | 94 | 100 | 96 | 97-98 |
| \#4 | 107 | 102 | 108 | 104 | 105-106 |
| \#9 | 115 | 110 | 116 | 112 | 113-114 |
| \#11 | 123 | 118 | 124 | 120 | 121-122 |
| \#15 | 131 | 126 | 132 | 128 | 129-130 |
| \#22 | 139 | 134 | 140 | 136 | 137-138 |
| \#30 | 147 | 142 | 148 | 144 | 145-146 |
| \#38 | 155 | 150 | 156 | 152 | 153-154 |
| \#50 | 163 | 158 | 164 | 160 | 161-162 |
| \#52 | 171 | 166 | 172 | 168 | 169-170 |
| \#71 | 179 | 174 | 180 | 176 | 177-178 |
| \#82 | 187 | 182 | 188 | 184 | 185-186 |
| \#101 | 197 | 192 | 198 | 194 | 195-196 |
| \#102 | 205 | 200 | 206 | 202 | 203-204 |
| \#104 | 213 | 208 | 214 | 210 | 211-212 |
| \#105 | 221 | 216 | 222 | 218 | 219-220 |
| \#106 | 229 | 224 | 230 | 226 | 227-228 |
| \#107 | 237 | 232 | 238 | 234 | 235-236 |
| \#108 | 245 | 240 | 246 | 242 | 243-244 |
| \#109 | 253 | 248 | 254 | 250 | 251-252 |
| \#110 | 263 | 258 | 264 | 260 | 261-262 |
| \#111 | 271 | 266 | 272 | 268 | 269-270 |
| \#112 | 287 | 282 | 288 | 284 | 285-286 |
| \#113 | 279 | 274 | 280 | 276 | 277-278 |
| \#114 | 295 | 290 | 296 | 292 | 293-294 |
| \#116 | 305 | 300 | 306 | 302 | 303-304 |
| \#118 | 313 | 308 | 314 | 310 | 311-312 |
| \#119 | 321 | 316 | 322 | 318 | 319-320 |

Congratulations - you are now ready to tackle any Math Olympiad problem!
Guide Page 3

MOEMS Success Rate: 84\%
MOEMS Problem Type: Circles, Cycling numbers
Solution Strategy: Diagram reasoning
MOEMS Problem Number: 3-2-2A
Problem \#9: An ant travels around the circle in the direction shown. It touches each of the labeled points in order. The first three points that the ant touches are A, B, and C, in that order. What is the 28th point that the ant touches?


## MOEMS Solution

METHOD 1: Strategy: Count by complete circuits of the circle.
The letter E is touched every 5 points beginning with the fifth point. Thus E is the $25^{\text {th }}$ touch and $\mathbf{C}$ is the $\mathbf{2 8}^{\text {th }}$ point the ant touches.
METHOD 2: Strategy: Count by individual points.
The points in order are ABCDE ABCDE ... . The $28^{\text {th }}$ point that the ant touches is C .

## Problem Assessment

Filled out for problem taught in September.

| Attribute | $\mathbf{5}^{\text {th }}$-Grade Rating | Potential Roadblocks |
| :--- | :--- | :--- |
| Math vocabulary | Known |  |
| Math concepts | Known |  |
| Wording | Simple |  |
| Translation | Simple |  |
| Solution strategy | Known |  |

Note: Because all five attributes are rated "known" or "simple," no potential roadblocks are listed for this problem.

Problem Assessment
for problem to be taught on $\qquad$ .

| Attribute | Your Class Rating | Potential Roadblocks |  |
| :--- | :--- | :--- | :--- |
| Math vocabulary | Known | New |  |
| Math concepts | Known | New |  |
| Wording | Simple | Complex |  |
| Translation | Simple | Complex |  |
| Solution strategy | Known | New |  |

## Plans to address roadblocks:

## Notes for next time

I taught the lesson on $\qquad$ .

1. Overall, I felt the lesson went...
2. I adjusted the Problem Assessment as..
3. Potential roadblocks I need to consider next time...
4. Other questions I could ask...
5. I had the students work (circle all that apply and comment)
4) Alone
5) With a learning partner
6) In a small group
6. Next time I teach this problem...

## Correct Solution Strategies

| A: | This "pinwheel" approach was a common strategy. It provided a straightforward counting solution to the problem. Those who counted correctly found that C is the $28^{\text {th }}$ point the ant touches. <br> The solution strategies of Mathletes B, C and $D$ show an evolution of mathematical growth. All three mathletes listed the five points and then proceeded to count to 28 . <br> Mathlete B listed each number; Mathlete C counted by fives; Mathlete D did the division and used only the remainder to count to letter C . |
| :---: | :---: |



## Sample Questions to Extend Mathlete Learning

- Describe your thinking in solving the problem.
- If we wanted to know the $68^{\text {th }}$ point touched is there an efficient way to figure it out?
- Did you try another way to solve the problem?
- How did you check your work?


## Notes for next time:

## Partial Solution Strategies

| E: | Mathlete E has written an open sentence <br> indicating they know what they are trying to <br> find. However, the mathlete made counting <br> errors (for example, skipping 15), revealing a <br> drawback in the solution strategy of writing <br> every number. <br> Sample Questions to Get Unstuck |
| :--- | :--- |
| - |  |
| Describe your thinking in solving the |  |
| problem. |  |
| - |  |
| Look at the numbers at point A. |  |
| Do you see a pattern? |  |

Notes for next time:

## Failed Start

This was the total extent of work on this mathlete's page. It
appears Mathlete $\mathbf{G}$ understood that the ant would move from
A to B to C and so on, but it seems they thought it stopped at
E , or perhaps they ran out of time.

## Sample Questions to Lead to Understanding

- What do we need to find? Write an open sentence.
- How is the ant traveling?
- What does "clockwise" mean?
- What is the label of the starting point?
- How many letters are there in the circle? What are there the letters?
- What is the last letter in each circle?
- How many points does the ant need to touch?
- How many full circles does the ant need to make?
- How many points in all has the ant touched in the 5 trips around?
- How many more letters must the ant touch?
- What are the next three letters?
- What is the last point the ant will touch?


## Notes for next time:

The KEV to Solving Mathematical Problems:
Stop! Think...Go...Think...Go...Think...Go...

Problem \#9: An ant travels around the circle in the direction shown. It touches each of the labeled points in order. The first three points that the ant touches are $A, B$, and $C$, in that order. What is the 28th point that the ant touches?


Answer

## Solution Strategy Worksheet for Problem \#9

Problem \#9: An ant travels around the circle in the direction shown. It touches each of the labeled points in order. The first three points that the ant touches are A, B, and C , in that order. What is the 28th point that the ant touches?


## STOP! Write an open sentence:

Think...Go...Think...Go...Think...Go...

1. Mathletes A and B used similar strategies to obtain a correct solution.
a. What did Mathlete A do?
b. What did Mathlete B do?
c. Which strategy is more efficient? Why?
2. Mathletes C and D also used similar strategies, which one is incorrect solution?
a. What did Mathlete C do? Is the answer correct? If not, why not?
b. What did Mathlete D do? Is the answer correct? If not, why not?
c. For the incorrect answer, where was the error in thinking?


Now REVIEW YOUR WORK:

- If you solved the problem correctly, can you improve your strategy to be more efficient?
- If you did not get the correct answer, can you find your error and correct your work?


## Answers to Solution Strategy Worksheet for Problem \#9

Problem \#9: An ant travels around the circle in the direction shown. It touches each of the labeled points in order. The first three points that the ant touches are A, B, and C, in that order. What is the 28th point that the ant touches?


STOP! Write an open sentence:
[28 ${ }^{\text {th }}$ point $\left.=\ldots.\right]$
Think...Go...Think...Go...Think...Go...

1. Mathletes A and B used similar strategies to obtain a correct solution.
a. What did Mathlete A do?
[A made a table with all the points in the top row and all the numbers from 1 to 28 below.]
b. What did Mathlete B do?
[B made a table with all the points in the top row, counted by five to 25 , and then listed the last three numbers.]
c. Why is strategy B more efficient than strategy A?
[Counting by fives is faster than counting by ones.]
2. Mathletes C and D also used similar strategies. Which one is an incorrect solution?
a. What did Mathlete C do?
[Mathlete $C$ divided 5 into 28 to get 5, remainder 3, and then listed the points $A, B, C$, $D, E$ and counted the remainder of 3 points, starting with A.]
Is the answer correct? [Yes.]
b. What did Mathlete D do?
[Mathlete D divided 5 into 28 to get 5, remainder 3 and then counted steps away from A, without counting point A itself.]

Is the answer correct? [No.]
c. For the incorrect answer, where was the error in thinking? [Mathlete $G$ reasoned that after touching all five points five times, the ant would end on point $A$, but the ant would actually be on $E$ at that moment. The ant doesn't reach $A$ again until the beginning of the $\sigma^{\text {th }}$ time around.]

# Mastering Word Problems <br> Using The Big Yellow Book (BYB) <br> BYB $=$ Big Yellow Book 

## Successfully solving math word problems requires both training and setting goals:

- Training: Learning how to analyze and persevere in solving complex math word problems.
- Goal: Successfully solve complex word problems.


## The Big Yellow Book is both an instructional guide for parents and a workbook for kids.

Scan to view video of students talking about learning from their mistakes.

- Instructional guide: Teaching Mathematical Problem Solving Using MOEMS ${ }^{\circledR}$ Contest Problems -- The Big Yellow Book -- provides step-by-step instructions for solving 28 carefully selected problems arranged in order of increasing complexity.
- Practice problems: Math Olympiad Contest Problems Volume 1, 2 or 3.
- Each Volume is a collection of fun and challenging word problems with solutions. You do not need to be affiliated with Math Olympiad to benefit from using these materials.
- All books are ordered from Math Olympiads at store.moems.org

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& \text { and to view the author's } \\
& \text { instructional videos } \\
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