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The *KEY*: An Empowering Routine for Solving Complex, Multistep Problems

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The *KEY* to Solving Mathematical Problems is a routine developed as the result of a thorough analysis of how students successfully persevere in solving complex, multistep problems.

Stop! Think. . .Go. . .Think. . .Go. . .Think. . .Go. . .

Each word in the *KEY* and the ellipses (. . .) direct students in the problem-solving process:

***Stop*!**	Write an open sentence so you know what you are solving for.
Think	Ask and answer questions that will help you understand the ideas in the problem.
Go	Try things, translate words into numbers, experiment.
. . .	Pause, slow down, and then move on.

This routine gives students permission to struggle with a problem. The *Stop!* tells students that they must be clear on what they are solving for before they pick up their pencil. The repeated *Think. . .Go. . .* reminds students to keep doubling back and thinking about the problem some more as they work on it, to ensure that their work is relevant and captures all of the nuances of the problem. The ellipses in the *KEY* conveys to students that they must slow down, and even pause, because perseverance is a natural part of the problem-solving process.

The *KEY* is a manifestation of Mathematical Practice 1: Make sense of problems and persevere in solving them. Teachers who use the *KEY* have reported dramatic improvements in perseverance in problem solving!

Since problem solving is an iterative, repetitive, thoughtful process, having a routine will support student growth in problem solving. Many people cannot read a complex, multistep problem only once and then solve it. They must go back, think, try things, and do it again. That is what the *KEY* assists in doing and is what mathematicians do when they solve problems.

The goal is for students to use the *KEY* with **every** problem. For maximum value, students must internalize the *KEY* and use it naturally. To make the *KEY* their automatic strategy requires behaviors very different from the approaches most teachers have reported over the years; namely, when given a word problem, students pick up their pencils and begin writing, using whatever numbers are given in the problem. Most students have had little experience with a problem that requires a longer period of time to solve. This is the rationale for practicing the approach: *Think. . .Go. . . Think. . .Go. . .* Unlike students' previous experiences, learning to pause and use the time to think and experiment with the problem is vital to finding a successful solution and developing confidence in their ability to solve problems.

To assist students in their transition to the *KEY* as their preferred, automatic problem-solving routine, there is a *KEY* Training Worksheet for them to use. ***Figure 1*** displays the *KEY* Training Worksheet for the problem: "The area of a rectangle is 36 sq cm. The lengths of the sides are whole numbers of centimeters. What is the largest perimeter the rectangle can have?" (answer: 74 cm).

The *KEY* Training Worksheet directs students to do a self-assessment as they work through a problem. This assessment is based on five problem attributes—vocabulary, math concepts, wording, translation, and strategy—that emerged as a result of the analysis of thousands of 4th and 5th grade student answers to complex, multistep Math Olympiad problems over a ten-year period. The *KEY* Training Worksheet includes a brief description of each attribute; a detailed presentation

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is in the book *Teaching Mathematical Problem Solving* (Frand and Sabean 2018).

Nicole Wilson, 5th Grade Teacher at La Ballona Elementary, Culver City, CA described the advantage of using a *KEY* Training Worksheet. She reported that giving her students the Problem Assessment "gave them the vocabulary they needed to explain to me what they didn't understand about the problem." This report is especially significant as La Ballona is a Title I school with a 65% Hispanic population and 39% are ESL students.

The *KEY* Training Worksheet is divided into two columns and contains steps that follow the *KEY*:

Step 1 directs students to *Stop* and write an open sentence.

Step 2 asks students to *Think* and assess the difficulty of the problem for themselves using the five problem attributes: vocabulary, math concepts, wording, translation, and strategy.

Step 3 directs students to *Go* solve the problem and *Think* about the problem attributes as they do so.

Students are encouraged to work back and forth between Steps 2 and 3 as they complete

The *KEY* to Solving Mathematical Problems:
Stop! Think. . . Go. . .Think. . .Go. . .Think. . .Go. . .
***KEY* Training Worksheet**

Problem: The area of a rectangle is 36 sq cm. The lengths of the sides are whole numbers of centimeters. What is the largest perimeter the rectangle can have?

Step 1: *Stop!* Write an open sentence so you know what you are solving for.

Step 2: *Think. . .*
What makes this problem hard?
As you solve this problem:

Think about what best describes your understanding of each statement. There are no right or wrong choices. Please be as honest with yourself as you can.

1. I understand the **vocabulary** in the problem:
   __ all words
   __ most words
   __ some words
   __ no words
2. I understand the **math concepts** in the problem:
   __ all math ideas
   __ most math ideas
   __ some math ideas
   __ no math ideas
3. The **wording** of the problem is:
   __ easy to understand
   __ a little hard to understand
   __ very hard to understand
   __ did not understand at all
4. I can **translate** the words in the problem into math ideas:
   __ easily
   __ with some difficulty
   __ with great difficulty
   __ I cannot do it at all
5. I can think of a **strategy** to solve the problem:
   __ easily
   __ with some difficulty
   __ with great difficulty
   __ I cannot do it at all

Step 3: *Go. . .*
As you work on the problem, for each statement at the left, select the choice that best describes you.

Figure 1: KEY Traning Worksheet

the problem. The focus is on the process of thinking about and understanding a problem while they work on solving the problem.

To successfully transition their students to the *KEY*, teachers need to use the *KEY* Training Worksheet over a period of several weeks and with a variety of problems. It also requires consistent use and reassurance from teachers for students to adopt this new approach to solving complex, multistep problems.

Most students have never written an open mathematics sentence nor have they done a self-assessment. To assist students in mastering how to use the *KEY*, dedicated lessons are taught for each aspect of the routine. For example, one lesson instructs students to evaluate open mathematics sentences to determine which are most efficient and effective for a given problem. Another lesson focuses on the five problem attributes so that students begin to use the terms to express where they are stuck in the problem-solving process. The book *Teaching Mathematical Problem Solving* expands on each of these ideas and has sample lesson plans for introducing the KEY.

Janet Lee, an upper-grade teacher at The Center for Early Education, a private K–6th grade school in Los Angeles summarized the value of the *KEY*. She stated: "My kids really grasped the clearly detailed process of *Stop! Think. . .Go. . .Think. . .Go. . .* and the purpose of each step. They knew they had to write an open sentence that helped them determine what they were solving for, and they learned to ask themselves questions to ascertain what made the problem challenging. It is that metacognitive process that really helped clarify their thinking."

Mary Tate, teacher and math coach at Ninth Street School in downtown Los Angeles, where 93% of her students are from low–income families, captured the breadth and reach of the KEY: "When I started training my 'gifted' students, I noticed that vocabulary was vital to the students' ability to tackle a problem. To add to this, when this vocabulary is used in certain syntax that was comprehensible to them, it made sense to them, and eventually helped them gain some success . . . They not only garnered skills, they also developed self-esteem and hope."

The *KEY* to Solving Mathematical Problems—*Stop! Think. . .Go. . .Think. . .Go. . . Think. . .Go. . .*—is a routine that makes the powerful ideas of George Polya accessible to students. Polya is "The Father of Problem Solving in Mathematics Education" according to the California Mathematics Council's website (CMC), and launched my problem-solving journey when I purchased a copy of his book, *How to Solve It*, as a mathematics student in 1964. Teachers have used Polya's ideas ever since they were published 75 years ago, but with varying degrees of success. For example, one approach uses a worksheet divided into quadrants labeled "What," "Plan," "Do," "Check." The center of the worksheet is a circle with a statement of the problem. The power of the *KEY* is that it interprets Polya's phases (steps) into language that is helpful to teachers in successfully introducing his ideas to their students.

The *KEY* empowers students to persevere while they engage in the process of solving complex, multistep problems. With experience, each student will develop his/her own problem-solving style. The value of the *KEY* is not for a particular type of problem nor a specific student's learning style. Rather, it is a generic problem-solving routine that can be applied in both mathematical and non-mathematical situations.

Video Note: To see a video of Jason demonstrating the introduction to the KEY to his class, please go to **www.moems.org/TMPS**.

References

California Mathematics Council (CMC). Infinity Wall: George Polya. Retrieved from **https://www.cmc-math.org/george-polya**.

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